education equals

Topic: Arithmetic and Geometric Sequences Applications

Time: 45 mins Marks: /45 marks

No calculator allowed

Question One: [2, 2, 1: 5 marks]

Sam invests $500 into a simple interest account. Sam is trying calculate how long it will take to doubles his money if he leaves it in this account. He begins generating the table below.

Year	Amount	Interest	Total
1	500	25	525
2	525	25	
3		25	
4			

a) Describe the recursive rule which could be used to describe the amount of money in the account at the start of each year.

b) How long will it take for Sam to double his money?

c) How does doubling the interest rate effect the situation?

Question Two: [2, 2: 4 marks]

Heather is calculating how many ancestors she has if she goes back each generation. If she looks one generation back she has her two parents, two generations back she has her four grandparents.

a) List the number of ancestors Heather has in each generation going back four generations.

b) Write a recursive formula for the number of ancestors Heather has if we go back n generations.

Question Three: [2, 2, 1: 5 marks]

A taxi charges a set flag fall and a charge per whole kilometre which is added together and charged to the passenger.

Eloise and Candy are meeting at the airport to go on holiday together. Eloise's taxi fare for the 6 km journey to the airport costs her $38. Candy lives 8 km from the airport and her taxi fare was $50.

a) How much does the taxi charge per kilometer?

b) What is the flag fall amount?

A third friend, Mary is also meeting the girls at the airport. Mary lives 3 kilometres from the airport.

c) How much is Mary charged for her trip to the airport?

Question Four: [2, 3, 2: 7 marks]

a) A ball is dropped from 1 metre above the ground. The ball hits the ground and then it bounces up 85% of the previous height. This pattern continues.

Write a recursive rule to describe the subsequent height of each bounce.

b) Nelson invests $300 in an account which is earning 12% interest per annum, compounding monthly. At the end of each month after the interest has been added he deposits an additional $90 into the account. Write a recursive rule to describe how his account is changing each month.

c) An electrician charges a $50 call out fee and then $80/hour charged in 30 minute blocks. Write a recursive rule to describe his charges for every 30 minute block of work.

Question Five: [3, 2, 2: 7 marks]

Julia is starting a social club. The first week she forgets to tell anyone about it so she is the only person who comes, the next week she brings one friend. The week after her and her friend each bring one new person. This pattern continues where each week the people who attended the previous week return and each bring with them a new person.

a) Draw a graph to show the number of people attending over the first six weeks.

b) Write the rule to find the number of people who would attend the n^{th} week of the social club.

c) The maximum capacity of the room in which they have their social club is 100 people. In which week do they reach the capacity of the room and how many people do they have to turn away that week?

Question Six: [3, 2, 2: 7 marks]

A venue has the hire price structure for a sit down function, as described below.

A fixed minimum cost of $400 for up to 72 people which includes 8 tables (each seating exactly 9 people). Additional people may be included in the booking but only in groups of 9 to complete a full table. Each additional person in this instance will cost $30 per person. The venue holds a maximum capacity of 135 people.

a) Complete table following table where, H_1 is the minimum hire with no additional tables, H_2 includes one additional table, H_3 includes two additional tables, and so on.

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Level of hire H_n	H_1	H_2	H_3	H_4	H_5	H_x
Number of guests G_n	72					135
$ cost of venue hire C_n	400					

b) Write a rule for the number of guests for each level of hire.

c) Write a recursive rule that will calculate the cost of venue hire.

Question Seven: [6, 2, 2: 10 marks]

Consider a sequence which is generated as follows:

$T_{n+2} = T_{n+1} + 2x - 3,\ T_2 = 3x - 1$

a) Write simplified expressions for the first four terms of the sequence.

b) Is this a geometric, arithmetic or linear sequence? Justify your answer.

c) If $2T_1 = T_2 + 2$ calculate the value of x.

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Topic: Arithmetic and Geometric Sequences Applications
SOLUTIONS

Time: 45 mins Marks: /45 marks

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Question One: [2, 2, 1: 5 marks]

Sam invests $500 into a simple interest account. Sam is trying calculate how long it will take to double his money if he leaves it in this account. He begins generating the table below.

Year	Amount	Interest	Total
1	500	25	525
2	525	25	550
3	550	25	575
4	575	25	600

a) Describe the recursive rule which could be used to describe the amount of money in the account at the start of each year.

$T_{n+1} = T_n + 25 \quad T_1 = 500$ ✓ ✓

b) How long will it take for Sam to double his money?

$4\ years = \$100$ ✓

$20\ years\ to\ add\ an\ extra\ \$500$ ✓

c) How does doubling the interest rate effect the situation?

Halves the time it takes to double his money. ✓

Question Two: [2, 2: 4 marks]

Heather is calculating how many ancestors she has if she goes back each generation. If she looks one generation back she has her two parents, two generations back she has her four grandparents.

a) List the number of ancestors Heather has in each generation going back four generations.

Number of Generations back	1	2	3	4
Number of Ancestors	2	4	8	16

✓ ✓

b) Write a recursive formula for the number of ancestors Heather has if we go back n generations.

✓ ✓

$T_n = 2 \times T_{n-1}$, $T_1 = 2$

Question Three: [2, 2, 1: 5 marks]

A taxi charges a set flag fall and a charge per whole kilometre which is added together and charged to the passenger.

Eloise and Candy are meeting at the airport to go on holiday together. Eloise's taxi fare for the 6 km journey to the airport costs her $38. Candy lives 8 km from the airport and her taxi fare was $50.

a) How much does the taxi charge per kilometer?

$$8d + f = 50$$

$$6d + f = 38$$

$$2d = 12 \quad ✓$$

$$d = 6$$

$$\therefore \$6/km \quad ✓$$

b) What is the flag fall amount?

$$6 \times 6 = 36 \quad ✓$$

$$+2 = 38$$

$$\$2 \; is \; flagfall \quad ✓$$

A third friend, Mary is also meeting the girls at the airport. Mary lives 3 kilometres from the airport.

c) How much is Mary charged for her trip to the airport?

$$6 \times 3 + 2 = \$20 \quad ✓$$

Question Four: [2, 3, 2: 7 marks]

a) A ball is dropped from 1 metre above the ground. The ball hits the ground and then it bounces up 85% of the previous height. This pattern continues.

Write a recursive rule to describe the subsequent height of each bounce.

$T_n = 0.85 \times T_{n-1}$, $T_1 = 0.85$

✓ ✓

b) Nelson invests \$300 in an account which is earning 12% interest per annum, compounding monthly. At the end of each month after the interest has been added he deposits an additional \$90 into the account. Write a recursive rule to describe how his account is changing each month.

$T_{n+1} = 1.01T_n + 90,$ $T_0 = 300\ \{or\ T_1 = 300 \times 1.01 + 90 = 393\}$

✓ ✓ ✓

c) An electrician charges a \$50 call out fee and then \$80/hour charged in 30 minute blocks. Write a recursive rule to describe his charges for every 30 minute block of work.

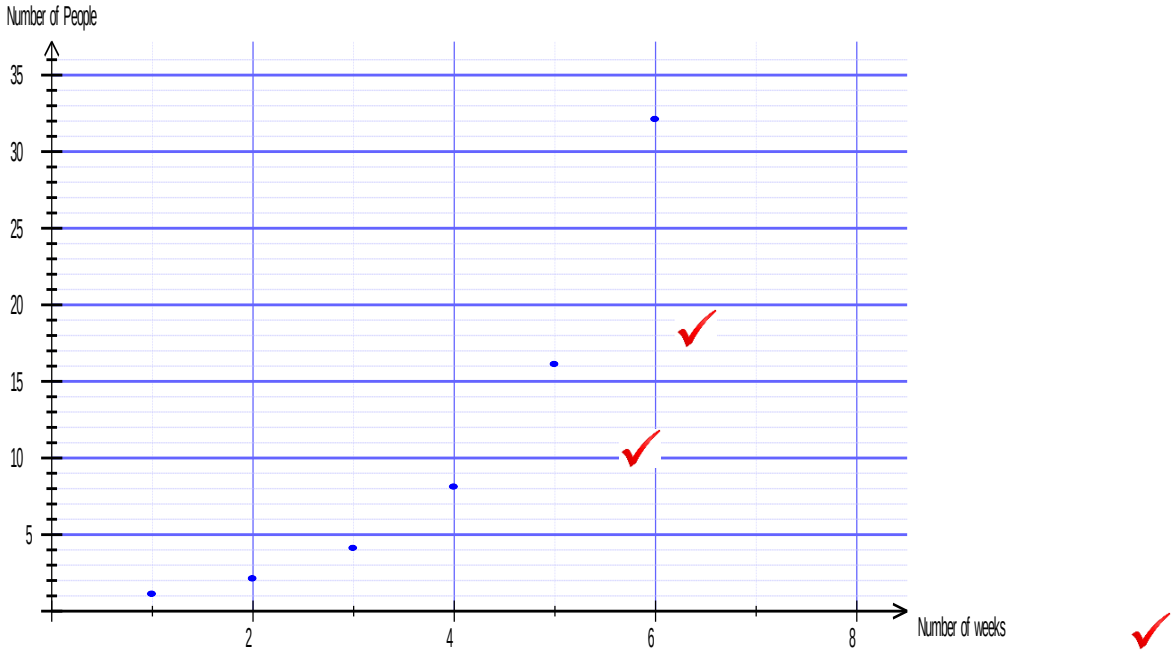
$T_n = 40 + T_{n-1},\ T_0 = 50\ \{or\ T_1 = 90\}$

✓ ✓

Question Five: [3, 2, 2: 7 marks]

Julia is starting a social club. The first week she forgets to tell anyone about it so she is the only person who comes, the next week she brings one friend. The week after her and her friend each bring one new person. This pattern continues where each week the people who attended the previous week return and each bring with them a new person.

a) Draw a graph to show the number of people attending over the first six weeks.



b) Write the rule to find the number of people who would attend the n^{th} week of the social club.

$T_n = 1 \times 2^{n-1}$

c) The maximum capacity of the room in which they have their social club is 100 people. In which week do they reach the capacity of the room and how many people do they have to turn away that week?

$T_8 = 128$

In the 8^{th} week they have to turn away 28 people.

Question Six: [3, 2, 2: 7 marks]

A venue has the hire price structure for a sit down function, as described below.

A fixed minimum cost of $400 for up to 72 people which includes 8 tables (each seating exactly 9 people). Additional people may be included in the booking but only in groups of 9 to complete a full table. Each additional person in this instance will cost $30 per person. The venue holds a maximum capacity of 135 people.

a) Complete table following table where, H_1 is the minimum hire with no additional tables, H_2 includes one additional table, H_3 includes two additional tables, and so on.

Level of hire	H_1	H_2	H_3	H_4	H_5	H_x
Number of guests	72	81	90	99	108	135
$ cost of venue hire	400	670	940	1210	1480	2290 ✓

✓ ✓

b) Write a rule for the number of guests for each level of hire.

$$G_n = 72 + 9(n-1)$$

✓ ✓

c) Write a recursive rule that will calculate the cost of venue hire.

$$C_{n+1} = C_n + 270\,, \qquad C_1 = 400$$

✓ ✓

Question Seven: [6, 2, 2: 10 marks]

Consider a sequence which is generated as follows:

$T_{n+2} = T_{n+1} + 2x - 3,\ T_2 = 3x - 1$

a) Write simplified expressions for the first four terms of the sequence.

$T_1 = 3x - 1 - (2x - 3)$ ✓

$= x + 2$ ✓

$T_2 = 3x - 1$

$T_3 = 3x - 1 + 2x - 3$ ✓

$= 5x - 4$ ✓

$T_4 = 5x - 4 + 2x - 3$ ✓

$= 7x - 7$ ✓

b) Is this a geometric, arithmetic or linear sequence? Justify your answer.

Arithmetic because there is a constant difference.
✓ ✓

c) If $2T_1 = T_2 + 2$ calculate the value of x.

$2(x + 2) = 3x - 1 + 2$ ✓

$2x + 4 = 3x + 1$

$x = 3$ ✓